From primordial ^{4}He abundance to the Higgs field.

Josef M. Gaßner, Harald Lesch

University Observatory Munich, Scheinerstr. 1, 81679 Munich, Germany

and

Hartmuth Arenhövel

Institut für Kernphysik, Johannes Gutenberg-Universität, 55099 Mainz, Germany

ABSTRACT

We constrain the possible time variation of the Higgs vacuum expectation value (v) by recent results on the primordial ^{4}He abundance (Y_P). For that, we improve the analytic models of the key-processes in our previous analytic calculation of the primordial ^{4}He abundance. Furthermore, a new dependence of the deuteron binding energy on v is incorporated, based on different nucleon-nucleon-potential-models.

Finally, we approximate the weak freeze-out, the cross section of photo-disintegration of the deuteron, the mean lifetime of the free neutron, the mass difference of neutron and proton, the Fermi coupling constant, the mass of the electron and the binding energy of the deuteron by terms of v, to constrain its possible time variation by recent results on the primordial ^{4}He abundance: $\left|\frac{\Delta v}{v}\right| \leq 1.5 \cdot 10^{-4}$.

Subject headings: cosmology: theory — cosmology: cosmological parameters — cosmology: early universe

1. Introduction

The standard model (Griffiths 1987) is a remarkably successful description of fundamental particle interactions. The theory contains parameters - such as particle masses - whose origins are still unknown and which cannot be predicted, but whose values are constrained through their interactions with the so called Higgs field. The Higgs field is assumed to have a non-zero value in the ground state of the universe - called its vacuum expectation value v - and elementary particles that interact with the Higgs field obtain a mass proportional to

this fundamental constant of nature.
Although the question whether the fundamental constants are in fact constant, has a long history of study (see Uzan (2003) for a review), comparatively less interest has been directed towards a possible variation of v (Gaßner & Lesch 2008; Chamoun et al. 2007; Landau et al. 2006; Li & Chu 2006; Yoo & Scherrer 2003; Ichikawa & Kawasaki 2002; Kujat & Scherrer 2000; Scherrer & Spergel 1993; Dixit & Sher 1988).
A macroscopic probe to determine the allowed variation range is given by the network of nuclear interactions during the Big-Bang-Nucleosynthesis (see Yao et al. (2006) for a review of the Standard Big-Bang-Nucleosynthesis Model), with its final abundance of ^{4}He. The relevant key-parameters are the freeze-out concentration of neutrons and protons, the so called deuterium bottleneck (the effective start of the primordial nucleosynthesis) and the neutron decay. Their dependencies on v and the final impact on the resulting primordial abundance of ^{4}He can be understood more clearly by an analytic approach.

Here we present a revised analytic calculation of the primordial ^{4}He abundance (Gaßner & Lesch 2008), where the analytic models of all key-processes have been improved. The opening of the deuterium bottleneck and the weak freeze-out are determined more accurate and a new dependence of the deuteron binding energy on v is incorporated, based on different nucleon-nucleon-potential-models.
The analytic approach enables us to take important issues into consideration, that have been ignored by previous works, like the v-dependence of the relevant cross sections of deuteron production and photo-disintegration. Furthermore, we take a non-equilibrium Ansatz for the freeze-out concentration of neutrons and protons and incorporate the latest results on the neutron decay and the baryon to photon ratio.

Finally, we approximate the weak freeze-out, the cross section of photo-disintegration of the deuteron, the mean lifetime of the free neutron, the mass difference of neutron and proton, the Fermi coupling constant, the mass of the electron and the binding energy of the deuteron by terms of v, to constrain its possible time variation by recent results on the primordial ^{4}He abundance (Peimbert et al. 2007; Izotov et al. 2007).

We briefly note, that constraints on the spacial variation of v required a measurement of helium abundance anisotropy or inhomogeneity versus the position in the sky and an inhomogeneous theoretical BBN model. The homogeneous formalism used throughout the paper thus assumes a spacial invariance of the Higgs vacuum expectation value.

2. Calculations

All relevant processes of SBBN took place at a very early epoch, when the energy density was dominated by radiation, leading to a time-temperature relation for a flat universe:

$$t = \sqrt{\frac{90\hbar^3 c^5}{32\pi^3 k^4 G g_*}\frac{1}{T^2}}\ [s], \tag{1}$$

where c is the velocity of light, k the Boltzmann constant, G denotes the gravitational constant and $\hbar$ is the Planck constant divided by 2π. g_* counts the total number of effectively massless ($mc^2 \ll kT$) degrees of freedom, given by $g_* = \left(g_b + \frac{7}{8} g_f\right)$, in which g_b represents the bosonic and g_f the fermionic contributions at the relevant temperature.
At very high temperatures ($T \gg 10^{10}$K), the neutrons and protons are kept in thermal and chemical equilibrium by the weak interactions

$$\begin{aligned} n + e^+ &\rightleftarrows p + \bar{\nu}_e, \\ n + \nu_e &\rightleftarrows p + e^-\ and \\ n &\rightleftarrows p + e^- + \bar{\nu}_e, \end{aligned}$$

until the temperature drops to a certain level, at which the inverse reactions become inefficient. This so called "freeze-out"-temperature T_f and time t_f denote the start of the effective neutron beta decay.

In previous works, the rate of neutron to proton concentration at freeze-out was calculated, assuming chemical and thermal equilibrium:

$$\frac{n_n}{n_p}(T_f) = e^{-\frac{Q}{kT_f}}, \tag{2}$$

where Q denotes the energy difference of neutron and proton rest masses. In fact, the deviation from equilibrium at freeze-out is already significant. Hence, we have to derive non-equilibrium concentrations, where we fall back on detailed calculations of Mukhanov (2004):
The 4-fermion-interaction $a + b \rightarrow c + d$ can be calculated using the Fermi theory, where the differential cross section is given by

$$\frac{d\sigma_{ab}}{d\Omega} = \frac{1}{(8\pi)^2}\frac{|M|^2}{(p_a + p_b)^2}\sqrt{\frac{(p_c \cdot p_d)^2 - m_c^2\ m_d^2\ c^8}{(p_a \cdot p_b)^2 - m_a^2\ m_b^2\ c^8}}. \tag{3}$$

$(p_a \cdot p_b)$ and $(p_c \cdot p_d)$ denote the scalar products of the 4-momenta and the matrix element is given by:

$$|M|^2 = 16(1 + 3g_A^2)\ G_F^2\ (p_a \cdot p_b)(p_c \cdot p_d),$$

where $G_F \simeq 1.166371 \cdot 10^{-5}$ $[\frac{1}{GeV^2}]$ (Yao et al. 2006) denotes the Fermi coupling constant and $g_A = 1.2739$ (Abele et al. 2002) the axial vector coupling constant, respectively.

First, we consider the reaction $n + \nu_e \rightarrow p + e^-$ at the relevant temperatures around few MeV and below, where the nucleons are nonrelativistic:

$$\begin{aligned}
(p_n + p_\nu)^2 &\simeq m_n^2 c^4 \\
(p_n \cdot p_\nu) &= m_n c^2 \; \epsilon_\nu \\
(p_p \cdot p_e) &= m_p c^2 \; \epsilon_e \\
\sqrt{(p_p \cdot p_e)^2 - m_p^2 m_e^2 c^8} &\simeq m_p \; c^2 \; \epsilon_e \; \sqrt{1 - \left(\frac{m_e c^2}{\epsilon_e}\right)^2} = m_p \; c \; \epsilon_e \; v_e
\end{aligned}$$

where m_p, m_n and m_e denote the mass of the proton, neutron and electron, respectively, v_e is the velocity of the electron, ϵ_ν is the energy of the incoming neutrino and $\epsilon_e \simeq \epsilon_\nu + Q$ is the energy of the outgoing electron. Substituting all terms into eq. (3) we obtain:

$$\begin{aligned}
\frac{d\sigma_{n\nu}}{d\Omega} &= \frac{1}{(8\pi)^2} \; 16 \; (1 + 3g_A^2) \; G_F^2 \; \frac{m_n \epsilon_\nu m_p \epsilon_e}{m_n^2} \frac{m_p \epsilon_e \frac{v_e}{c}}{\sqrt{(m_n \epsilon_\nu)^2 - (m_n m_\nu c^2)^2}} \qquad (4) \\
&= \frac{1}{(2\pi)^2} \; (1 + 3g_A^2) \; G_F^2 \; \frac{m_p^2}{m_n^2} \; \epsilon_e^2 \; \frac{v_e}{c}, \qquad (5)
\end{aligned}$$

where we neglect the neutrino mass m_ν. Integration leads to

$$\sigma_{n\nu} = \frac{1 + 3g_A^2}{\pi} \; G_F^2 \; \frac{m_p^2}{m_n^2} \; \epsilon_e^2 \; \frac{v_e}{c}. \qquad (6)$$

Next, we have to consider that at temperatures $kT > 2 \; m_e c^2$ the possible states for the electron are partially occupied by electron-positron-pairs. According to the Pauli exclusion principle, this reduces the appropriate cross section to

$$\sigma_{n\nu}^* = \sigma_{n\nu} \; \frac{1}{1 + e^{-\frac{\epsilon_e}{kT}}}. \qquad (7)$$

This enables us to calculate ΔN_n, the reduction of neutrons within time interval Δt in a given volume, containing N_n neutrons:

$$\Delta N_n = -\left(\sum_{\epsilon_\nu}^{\epsilon_\nu + \Delta\epsilon_\nu} \sigma_{n\nu}^* n_{\epsilon_\nu} v_\nu \Delta g_{\epsilon_\nu} \right) N_n \Delta t. \qquad (8)$$

where

$$n_{\epsilon_\nu} = \frac{1}{1 + e^{\frac{\epsilon_\nu}{kT_\nu}}} \qquad (9)$$

denotes the neutrino occupation number (v_ν and T_ν are the velocity and the temperature of the neutrinos) and

$$\Delta g_{\epsilon_\nu} = \frac{1}{2\pi^2} \int_{\epsilon_\nu}^{\epsilon_\nu+\Delta\epsilon_\nu} |p|^2 \; d|p| \simeq \frac{1}{2\pi} \; \epsilon_\nu^2 \; \Delta\epsilon_\nu \tag{10}$$

the phase volume element. Introducing the relative concentration of the neutrons

$$X_n = \frac{n_n}{n_n + n_p} \tag{11}$$

and assuming baryon conservation, we obtain the rate of change of the neutron concentration due to $n\nu$-process:

$$\left(\frac{dX_n}{dt}\right)_{n\nu} = -\lambda_{n\nu} X_n, \tag{12}$$

where $\lambda_{n\nu}$ denotes the decay rate. Substituting the cross section (7) into eq. (8) we obtain:

$$\lambda_{n\nu} = \frac{1+3g_A^2}{2\pi^3} \; G_F^2 \; \frac{m_p^2}{m_n^2} \; \Im(T_\nu), \tag{13}$$

where

$$\Im(T_\nu) = \int_0^\infty \epsilon_e^2 \sqrt{1 - \left(\frac{m_e c^2}{\epsilon_e}\right)^2} \; \frac{1}{1+e^{-\frac{\epsilon_e}{kT}}} \; \frac{\epsilon_\nu^2}{1+e^{\frac{\epsilon_\nu}{kT_\nu}}} \; d\epsilon_\nu. \tag{14}$$

Below the temperature $kT \simeq 2 \; m_e \; c^2$, the Pauli exclusion principle, represented by the term $(1+e^{-\frac{\epsilon_e}{kT}})$, looses importance and numerically we notice a deviation of 1 % only, when we set this term to 1. Expanding the square root ($m_e c^2/\epsilon_e \ll 1$) keeping only first two terms and introducing the integration variable $x = \frac{\epsilon_\nu}{kT_\nu}$ we derive:

$$\Im(T_\nu) = (kT_\nu)^5 \int_0^\infty x^2 \frac{\left(x + \frac{Q}{kT_\nu}\right)^2 - \frac{1}{2}\left(\frac{m_e c^2}{kT_\nu}\right)^2}{1+e^x} \; dx \tag{15}$$

$$= Q^5 \; \left(\frac{kT_\nu}{Q}\right)^3 \left[\frac{45\;\zeta(5)}{2}\left(\frac{kT_\nu}{Q}\right)^2 + \frac{7\pi^4}{60}\left(\frac{kT_\nu}{Q}\right) + \frac{3\;\zeta(3)}{2}\left(1 - \frac{m_e^2 c^4}{2\;Q^2}\right)\right] \tag{16}$$

$$\simeq Q^5 \; \frac{45\;\zeta(5)}{2} \; \left(\frac{kT_\nu}{Q}\right)^3 \left(\frac{kT_\nu}{Q} + 0.25\right)^2, \tag{17}$$

where ζ is the Riemann zeta function. In the last step, we completed the square approximately. Finally we convert $\lambda_{n\nu}$ from MeV to $\frac{1}{s}$ and derive

$$\lambda_{n\nu} \simeq \frac{1+3g_A^2}{1.75 \cdot 10^{-21}} \; G_F^2 \; \frac{m_p^2}{m_n^2} \; Q^5 \left(\frac{kT_\nu}{Q}\right)^3 \left(\frac{kT_\nu}{Q} + 0.25\right)^2 \; \left[\frac{1}{s}\right]. \tag{18}$$

Similarly, we find the decay rate of the reaction $n + e^+ \to p + \bar{\nu}$ (we interchange ϵ_ν with ϵ_e and m_e with $m_\nu = 0$):

$$\lambda_{ne} = \frac{1+3g_A^2}{2\pi^3}\; G_F^2\; \frac{m_p^2}{m_n^2} \int_{m_e c^2}^{\infty} \epsilon_\nu^2\; \frac{\epsilon_e^2}{1+e^{\frac{\epsilon_e}{kT}}}\; d\epsilon_e \tag{19}$$

Assuming $T_\nu = T$, the rates of the inverse reactions are related to the rate of the direct reactions as

$$\lambda_{pe} = e^{-\frac{Q}{kT}} \lambda_{n\nu} \tag{20}$$

$$\lambda_{p\nu} = e^{-\frac{Q}{kT}} \lambda_{ne}. \tag{21}$$

Hence, we can write the following balance equation for X_n:

$$\begin{aligned} \frac{dX_n}{dt} &= -(\lambda_{n\nu} + \lambda_{ne})X_n + (\lambda_{pe} + \lambda_{p\nu})(1 - X_n) \\ &= -(\lambda_{n\nu} + \lambda_{ne})(1 + e^{-\frac{Q}{kT}})(X_n - X_n^{eq}) \end{aligned} \tag{22}$$

with the equilibrium neutron concentration

$$X_n^{eq} = \frac{1}{1 + e^{\frac{Q}{kT}}}. \tag{23}$$

To solve this linear differential equation (22), we take the initial condition $X_n(t=0) = X_n^{eq}$ and obtain:

$$X_n(t) = X_n^{eq}(t) - \int_0^t \exp\left(- \int_{\tilde{t}}^t (\lambda_{n\nu}(y) + \lambda_{ne}(y))(1 + e^{-\frac{Q}{kT}})dy \right) \dot{X}_n^{eq}(\tilde{t}) d\tilde{t}, \tag{24}$$

where dot denotes the derivative with respect to time. Using the auxiliary function $F(t)$

$$F(t) = \int_0^t (\lambda_{n\nu}(t) + \lambda_{ne}(t))(1 + e^{-\frac{Q}{kT}})dt, \tag{25}$$

we express the integral in (24) in the form

$$\int_0^t e^{-F(t)+F(\tilde{t})}\; \dot{X}_n^{eq}(\tilde{t})\; d\tilde{t} \tag{26}$$

and expand in the small parameter $(t - \tilde{t})$, since the integral is dominated by the contribution of $\tilde{t} \simeq t$ if $F(t)$ is a quickly growing function of t:

$$X_n(t) = X_n^{eq}(t) - \int_0^t \left[\dot{X}_n^{eq}(t) + \ddot{X}_n^{eq}(t)(\tilde{t} - t) + ... \right]\; e^{-\dot{F}(t-\tilde{t})} \left(1 + \frac{1}{2}\ddot{F}(t)(\tilde{t} - t)^2 + ... \right) dt \tag{27}$$

We integrate term by term using

$$\int_0^t e^{-A(t-\tilde{t})}(t-\tilde{t})^n \; d\tilde{t} \simeq A^{-n-1} \; n!, \tag{28}$$

where we neglect exponentially small terms of order e^{-At}, deriving:

$$X_n(t) = X_n^{eq}(t) \left(1 - \frac{1}{(\lambda_{n\nu}+\lambda_{ne})(1+e^{-\frac{Q}{kT}})} \frac{\dot{X}_n^{eq}(t)}{X_n^{eq}(t)} + ... \right). \tag{29}$$

Later, when the temperature has dropped significantly, X_n^{eq} goes to zero and the integral in eq. (24) approches the finite limit. As a result, the neutron concentration freezes-out at $X_n(t \to \infty)$. Effectively, this freeze-out occurs, when the deviation from equilibrium becomes significant, hence when

$$\frac{\dot{X}_n^{eq}}{X_n^{eq}} \simeq (\lambda_{n\nu}+\lambda_{ne})(1+e^{-\frac{Q}{kT}}). \tag{30}$$

Assuming this happens before $e^{\pm}$-annihilation and after kT has dropped below Q, we set $\lambda_{n\nu}+\lambda_{ne} \simeq 2\lambda_{n\nu}$ and neglect the term $\exp(-Q/kT)$. Substituting all terms and taking the time-temperatur-relation (1) into account, we finally obtain an equation for the freeze-out-temperature T_f:

$$\sqrt{\frac{Gg_*(T_f)}{\hbar^3 c^5}} \; 3.7 \cdot 10^{-35} = (1+3g_A^2) \; G_F^2 \; Q^3 \; \left(\frac{kT_f}{Q}\right)^2 \left(\frac{kT_f}{Q}+0.25\right)^2. \tag{31}$$

The quadratic term $\frac{kT_f}{Q}$ leads to

$$T_f \simeq 1.16 \cdot 10^6 \; \frac{Q}{k} \; \left[-\frac{1}{8} + \sqrt{\frac{1}{64} + \sqrt{\frac{3.7 \cdot 10^{-35}}{(1+3g_A^2) \; G_F^2 \; Q^3}} \left(\frac{Gg_*(T_f)}{\hbar^3 c^5}\right)^{1/4}} \right] \; [K]. \tag{32}$$

At T_f the effectively massless species in the cosmic plasma are neutrinos (left-handed only), antineutrinos (right-handed only), electrons, positrons and photons. For the case of three neutrino families ($N_\nu = 3$), we obtain $g_*(T_f) = (2 + \frac{7}{8}(4+2N_\nu) = 10.75$.
To calculate the relevant neutron concentration at T_f, we go back to eq. (24). Since $X_n^{eq} \to 0$ as $T \to 0$ we have to calculate the integral term in eq. (24) in the limit $t \to \infty$. The main contribution to the integral comes at temperature above the restmass of the electron, where again $\lambda_{n\nu}+\lambda_{ne} \simeq 2 \; \lambda_{n\nu}$ ($\lambda_{n\nu}$ given by eq. (18)). Furthermore, we use eq. (1) to change the integration variable from dt to dT:

$$X_n(T_f) = \int_0^\infty \frac{Q \; \exp\left[-2.68 \cdot 10^{34} \sqrt{\frac{\hbar^3 c^5}{Gg_*(T_f)}} \; (1+3g_A^2) \; G_F^2 \int_0^T (x+\frac{Q}{4})^2 (1+e^{-\frac{Q}{x}}) dx\right]}{2 \; T^2 \left(1+\cosh\frac{Q}{T}\right)} dT, \tag{33}$$

with Q and T in units of MeV. For later purposes, we finally state the neutron to proton ratio at freeze-out:

$$\frac{n_n}{n_p}(T_f) = \frac{1}{\frac{1}{X_n(T_f)} - 1} \tag{34}$$

In comparison, the equilibrium rate eq. (2) is 14 % higher, thus as mentioned before, the deviation is significant, which justifies the effort.

From now on the loss of free neutrons via $n \to p+e^- +\bar{\nu}_e$, with a mean lifetime (Serebrov et al. 2005) $\tau_n = 878.5$ s, can no longer be refurbished. Thus, whereas the neutron density decreases as $n_n(t) = n_n(t_f) \cdot e^{-\frac{t-t_f}{\tau_n}}$, the proton density increases as $n_p(t) = n_p(t_f) + (n_n(t_f) - n_n(t))$ and we obtain

$$\frac{n_p}{n_n}(t) = \frac{e^{\frac{t-t_f}{\tau_n}}}{X_n(T_f)} - 1. \tag{35}$$

At the relevant densities in the early universe, fusion reactions can only proceed efficiently through sequences of two-body-collisions and the starting product of this collisions is the shallow bound deuteron ($B_d \simeq 2.225$ MeV), which is highly affected by photo-disintegration. The start of nucleosynthesis, t_N, is therefore usually referred to as the "deuterium bottleneck".

Once the deuteron production dominates the photo-disintegration and the expansion of the universe, our calculation in some sense "only produces deuteron", disregarding that deuteron is also destroyed by the fusion of light elements. In fact, we do not consider the detailed fusion reactions with its intermediate products that finally lead to ^{4}He. We are interested in the point t_N, when we can assume neutron conservation, because enough neutrons have reached stable states inside light nuclei (no matter whether inside deuterons or further fusion products of deuteron). Hence, we obtain t_N assuming two boundary conditions: First, the deuteron production must dominate the photo-disintegration and the expansion of the universe. Second, to justify neutron conservation, the deuteron density must exceed the density of free neutrons. In fact, it turns out, that we can reproduce the numerical as well as the observation based results on Y_P, assuming neutron conservation when $52 - 66$ % of the neutrons have reached stable states.
The interval between t_f and t_N is a substantial fraction of the neutron lifetime and therefore plays an essential role for the outcome of the primordial helium production. Thus, we have to calculate the rates of deuteron production $\Gamma_{(np\to d\gamma)}$, deuteron photo-disintegration $\Gamma_{(\gamma d\to np)}$ and the rate of deuteron reduction by the expansion of the universe $\Gamma_{expansion}$, to determine t_N respectively T_N, when

$$\Gamma_{(np\to d\gamma)} > \Gamma_{(\gamma d\to np)} + \Gamma_{expansion}. \tag{36}$$

The rates for production and photo-disintegration of deuteron are given by the product of the relevant number densities, velocities and cross sections, whereas the expansion rate of a radiation-dominated, flat universe is given by $\frac{1}{2t}$ with t from eq. (1), leading to:

$$n_n \frac{\eta\ n_\gamma}{1+\frac{n_n}{n_p}} \sqrt{\frac{8kT}{\pi m_N}}\ \sigma_{(np\to d\gamma)} > n_d\ n_\gamma^*\ c\ \sigma_{(\gamma d\to np)} + \frac{n_d}{2\ t}, \tag{37}$$

where $\sigma_{(\gamma d\to np)}$ denotes the cross section of deuteron photo-disintegration, $\sigma_{(np\to d\gamma)}$ the cross section of deuteron production, m_N is the nucleon mass, $\eta \simeq 6.226\cdot 10^{-10}$ is the baryon to photon ratio based on WMAP (Komatsu et al. 2008) and Steigman (2008), n_d and n_γ denote the number densities of deuterons and photons, respectively, and n_γ^* the number density of photons which supply enough energy to disintegrate the deuteron and do not loose this energy in much more likely Compton scattering on electrons.
The number density of photons at a certain temperature T is given by

$$n_\gamma = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{E_\gamma^2}{e^{\frac{E_\gamma}{kT}}-1} dE_\gamma = 16\pi\ \zeta(3)\left(\frac{kT}{hc}\right)^3, \tag{38}$$

where ζ is the Riemann zeta function. The number density of these photons supplying a minimum energy $E_\gamma > B_d \gg kT$ is

$$n_{(\gamma>B_d)} = \frac{8\pi}{(hc)^3}\int_{B_d}^\infty E_\gamma^2 e^{-\frac{E_\gamma}{kT}} dE_\gamma = 8\pi\left(\frac{kT}{hc}\right)^3\left[\left(\frac{B_d}{kT}+1\right)^2+1\right]e^{-\frac{B_d}{kT}}, \tag{39}$$

but most of them will loose energy in Compton scattering on electrons, leading to

$$n_\gamma^* = n_{(\gamma>B_d)}\ \frac{n_d\ \sigma_{(\gamma d\to np)}}{n_p\ \sigma_{(\gamma e\to e\gamma)}}, \tag{40}$$

where $\sigma_{(\gamma e\to e\gamma)}$ denotes the Klein-Nishina cross section (Rybicki & Lightman 1979) for Compton scattering on electrons:

$$\sigma_{(\gamma e\to e\gamma)} = \frac{1}{8\pi}\left(\frac{e^2}{\epsilon_0 m_e c^2}\right)^2\left[\frac{1+\beta}{\beta^2}\left(\frac{2(1+\beta)}{1+2\beta}-\frac{\ln(1+2\beta)}{\beta}\right)+\frac{\ln(1+2\beta)}{2\beta}-\frac{1+3\beta}{(1+2\beta)^2}\right] \tag{41}$$

where

$$\beta = \frac{\langle E_\gamma\rangle}{m_e c^2}, \tag{42}$$

and the mean incident photon energy $\langle E_\gamma\rangle$ is given by

$$\langle E_\gamma\rangle = \frac{1}{n_{(\gamma>B_d)}}\frac{8\pi}{(hc)^3}\int_{B_d}^\infty E_\gamma^3 e^{-\frac{E_\gamma}{kT}} dE_\gamma = kT\left[\frac{\left(\frac{B_d}{kT}\right)^3}{\left(\frac{B_d}{kT}+1\right)^2+1}+3\right] \simeq B_d + kT. \tag{43}$$

The interaction cross section of deuteron photo-disintegration can be well approximated by (Bethe & Longmire 1950):

$$\sigma_{BL} = E1 + M1 \tag{44}$$

where we express the electric dipole contribution

$$E1 = \frac{2}{3}\ \frac{e^2\ \hbar\ \sqrt{B_d}\ (E_\gamma - B_d)^{\frac{3}{2}}}{c\ \epsilon_0\ m_N\ E_\gamma^3\ \left(1 - \frac{r_t}{\hbar}\sqrt{m_N B_d}\right)} \tag{45}$$

and the magnetic dipole contribution

$$M1 = \frac{e^2\hbar(\mu_p - \mu_n)^2}{6\ \epsilon_0\ m_N^2\ c^3}\sqrt{\frac{B_d}{E_\gamma} - \left(\frac{B_d}{E_\gamma}\right)^2}\ \frac{\left(1 - \sqrt{m_N B_d}\frac{a_s}{\hbar} + a_s(r_s + r_t)\frac{m_N B_d}{4\hbar^2} - a_s(r_s - r_t)\frac{m_N(E_\gamma - B_d)}{4\hbar^2}\right)^2}{(1 + a_s^2\frac{m_N(E_\gamma - B_d)}{\hbar^2})(1 - \frac{r_t}{\hbar}\sqrt{m_N B_d})} \tag{46}$$

in terms of B_d and E_γ. ϵ_0 denotes the electric constant, a_s and a_t singlet and triplet scattering length, r_s and r_t singlet and triplet effective range, μ_p and μ_n the magnetic moment of proton and neutron, respectively.
With this dependence of σ_{BL} on a certain incident photon energy E_γ, we derive $\sigma_{(\gamma d \to np)}$ as the mean cross section of photo-disintegration per high-energetic photon ($E_\gamma > B_d$):

$$\sigma_{(\gamma d \to np)} = \frac{1}{n_{(\gamma > B_d)}} \frac{8\pi}{(hc)^3} \int_{B_d}^{\infty} E_\gamma^2\ e^{-\frac{E_\gamma}{kT}}\ \sigma_{BL}(E_\gamma)\ dE_\gamma. \tag{47}$$

We go back to eq. (37) and divide by $\Gamma_{(\gamma d \to np)}$, in order to receive two terms, that we analyse separately:

$$\frac{\Gamma_{(np \to d\gamma)}}{\Gamma_{(\gamma d \to np)}} = 1 + \frac{\Gamma_{expansion}}{\Gamma_{(\gamma d \to np)}}. \tag{48}$$

We start with

$$\frac{\Gamma_{(np \to d\gamma)}}{\Gamma_{(\gamma d \to np)}} = \frac{n_n\ \eta\ \sqrt{\frac{8kT}{\pi m_N}}\ \sigma_{(np \to d\gamma)}}{n_d\ \left(1 + \frac{n_n}{n_p}\right)\ \frac{n_\gamma^*}{n_\gamma}\ c\ \sigma_{(\gamma d \to np)}} \tag{49}$$

$$= \frac{3.84\ \eta\ \sqrt{\frac{kT}{m_N c^2}}\ e^{\frac{B_d}{kT}}\ \frac{n_p}{n_d}\ \frac{\sigma_{(\gamma e \to e\gamma)}}{\sigma_{(\gamma d \to np)}}}{\left(1 + \frac{n_n}{n_p}\right)\left[\left(\frac{B_d}{kT} + 1\right)^2 + 1\right]}\ \frac{\sigma_{(np \to d\gamma)}}{\sigma_{(\gamma d \to np)}} \tag{50}$$

where $\sigma_{(np \to d\gamma)}$ is related to $\sigma_{(\gamma d \to np)}$ by the detailed balance

$$\frac{\sigma_{(np \to d\gamma)}}{\sigma_{(\gamma d \to np)}} \simeq \frac{3\langle E_\gamma \rangle^2}{2 m_N c^2(\langle E_\gamma \rangle - B_d)}, \tag{51}$$

and $\langle E_\gamma \rangle$ is given by eq. (43), leading to

$$\frac{\Gamma_{(np\to d\gamma)}}{\Gamma_{(\gamma d\to np)}} = \frac{5.755\ \eta}{1+\frac{n_n}{n_p}} \left(\frac{kT}{m_N c^2}\right)^{\frac{3}{2}} e^{\frac{B_d}{kT}}\ \frac{\sigma_{(\gamma e\to e\gamma)}}{\sigma_{(\gamma d\to np)}}\ \left(\frac{n_n}{n_d}\right)^2 \frac{n_p}{n_n}. \tag{52}$$

Next, we analyse the term on the right hand side of eq. (48):

$$\frac{\Gamma_{expansion}}{\Gamma_{(\gamma d\to np)}} = \frac{\frac{n_d}{2\ t}}{n_d\ n_\gamma^*\ c\ \sigma_{(\gamma d\to np)}} \tag{53}$$

$$= 1.040 \sqrt{\frac{G g_*(T_N) h^3}{c}}\ \frac{kT\ e^{\frac{B_d}{kT}}}{(B_d+kT)^2}\ \frac{\sigma_{(\gamma e\to e\gamma)}}{\sigma^2_{(\gamma d\to np)}}\ \frac{n_p}{n_n}\ \frac{n_n}{n_d}. \tag{54}$$

Both analysed terms depend on T exponentially, but at $T = T_N$, when term(52) reaches unity, term(54) is still of order 10^{-2} and therefore negligible. In other words, the expansion rate of the universe is still dominated by the rate of deuteron photo-disintegration, when the primordial nucleosynthesis starts. Of course, the expansion of the universe is the motivation force to open the deuterium bottleneck, but its major influence is the reddening of the radiation, which enables the deuteron production to win the upper hand over photo-disintegration. This simplifies our task drastically and we obtain the following equation for T_N, respectively t_N:

$$\frac{5.755\ \eta}{1+\frac{n_n}{n_p}(T_N)} \left(\frac{kT_N}{m_N c^2}\right)^{\frac{3}{2}} e^{\frac{B_d}{kT_N}}\ \frac{\sigma_{(\gamma e\to e\gamma)}}{\sigma_{(\gamma d\to np)}}(T_N)\ \left(\frac{n_n}{n_d}(T_N)\right)^2 \frac{n_p}{n_n}(T_N) = 1 \tag{55}$$

and finally the corresponding neutron to proton ratio:

$$\frac{n_p}{n_n}(T_N) = \frac{1}{X_n(T_f)} \exp\left[\frac{1}{\tau_n}\sqrt{\frac{90\hbar^3 c^5}{32\pi^3 k^4 G}}\left(\frac{1}{\sqrt{g_*(T_N)}T_N^2} - \frac{1}{\sqrt{g_*(T_f)}T_f^2}\right)\right] - 1 \tag{56}$$

where X_n is given by eq. (33) .

The neutrinos have decoupled from equilibrium at about one MeV and thus before the annihilation of electron positron pairs. Therefore the entropy due to this annihilation is transferred exclusively to the photons, i.e. $g_*(T_N) = 2 + \ \frac{7}{8}\ 2\ N_\nu\ (\frac{4}{11})^{\frac{4}{3}}$.

Assuming neutron conservation after t_N, we finally calculate Y_P, the primordial ^{4}He abundance by weight. Since ^{4}He is not further transformed into heavier nuclei, because elements with nucleon mass number A=5 and A=8 are insufficiently stable to function successfully as intermediate products for nucleosynthesis at the available densities, we derive:

$$Y_P = \frac{\frac{1}{2}n_n m_{He}}{\frac{1}{2}n_n m_{He} + (n_p - n_n)m_p} = \frac{1}{1 + 2\frac{m_p}{m_{He}}\left(\frac{n_p}{n_n}(T_N) - 1\right)} \tag{57}$$

where m_{He} denotes the mass of the helium nucleus and $\frac{n_p}{n_n}(T_N)$ is given by eq. (56).

This analytic expression for Y_P reproduces the observation based results and the numerical results (see section results for details), assuming neutron conservation when $52 - 66$ % of the neutrons have reached stable states. Therefore eq. (57) provides our basis for finding the dependence of Y_P and the possible deviation of v from its present value v_0, in order to finally constrain $\frac{v}{v_0}$ by recent results on the primordial ^{4}He abundance.

Expressing all key-parameters of Y_P by terms of v, we start with the most important one, the deuteron binding energy. Within our narrow range of interest ($|\frac{v-v_0}{v_0}| < 0.5\%$), we determine its dependence on v by varying the pion mass in different nucleon-nucleon-potential-model calculations, based on (Arenhövel & Sanzone 1991), and derive:

$$B_d(v) \simeq B_d(v_0)\ (A - (A-1)\sqrt{v/v_0}), \tag{58}$$

where A is a model dependent constant as follows:

Bonn-A potential (Machleidt et al. 1987):	A =	2.3,
Paris potential (Lacombe et al. 1980):	A =	28.2,
Argonne V14 potential (Wiringa et al. 1984):	A =	61.4.

For our purpose, we take the mean average $A = 30.6$.

As B_d changes, E_γ and the cross sections $\sigma_{(\gamma d \rightarrow np)}$ and $\sigma_{(np \rightarrow d\gamma)}$ change, accordingly. Furthermore, we have to consider, that the mass of the electron varies proportionally

$$m_e(v) = m_e(v_0)\ \frac{v}{v_0}, \tag{59}$$

which enters the Klein-Nishina cross section.

Concerning τ_n, the mean lifetime of the free neutron, we use the expression (Gaßner & Lesch 2008), based on (Müller et al. 2004):

$$\tau_n(v) \simeq \tau_n(v_0)\ (1 - 4.88\ \frac{v - v_0}{v_0}). \tag{60}$$

Next, we have to consider the change on Q, the neutron to proton mass difference, which influences the freeze-out concentration. We separate the electromagnetic contribution (Gasser & Leutwyler 1982) and obtain

$$Q \simeq (-0.76 + 2.0533317\ \frac{v}{v_0})\ [MeV]. \tag{61}$$

The Fermi coupling constant G_F is related to v by (Dixit & Sher 1988):

$$G_F(v) = \frac{1}{v^2\sqrt{2}}\ \left[\frac{1}{GeV^2}\right]. \tag{62}$$

Finally, we derive a relation between Y_P and v, to constrain the permitted variation of the Higgs vacuum expectation value by the primordial ^{4}He abundance:

$$\begin{aligned}\Delta Y_P \simeq & -38\left(\frac{\Delta v}{v}\right)^2 - 2.08\left(\frac{\Delta v}{v}\right) + \\ & + 0.0410\left(\frac{N_\nu - 3}{3}\right) + 0.0874\left(\frac{\Delta G}{G}\right) + 0.0042\ \ln\left(\frac{\Delta \eta}{\eta}\right). \end{aligned} \qquad (63)$$

Varying each parameter separately (assuming the others fixed), we derive:

$$\begin{aligned}\Delta Y_P \simeq & \ 0.106\left(\frac{\Delta B_d}{B_d}\right) + 0.056\left(\frac{\Delta \tau_n}{\tau_n}\right) - 0.235\left(\frac{\Delta G_F}{G_F}\right) - 0.352\left(\frac{\Delta Q}{Q}\right) - 0.006\left(\frac{\Delta m_e}{m_e}\right) + \\ & + 0.0410\left(\frac{\Delta N_\nu}{3}\right) + 0.0874\left(\frac{\Delta G}{G}\right) - 0.2602\left(\frac{\Delta \hbar}{\hbar}\right) + 0.0042\ \ln\left(\frac{\Delta \eta}{\eta}\right). \end{aligned} \qquad (64)$$

We briefly note, that using eq. (63), one can also obtain constraints on N_ν, G, $\hbar$ and η.

3. Results

Constraining the possible time variation of v, we use observation based results as well as the standard big bang nucleosynthesis code, developed by Wagoner (1973) and Kawano (1992). This standard code still seems adequate for our purpose, although newer nuclear reaction rates have been evaluated (Descouvemont et al. 2004).
The baryon to photon ratio is given by $\eta = (273.9 \pm 0.3)\ \Omega_b h^2$ (Steigman 2008), where Ω_b is the present ratio of the baryon mass density to the critical density and h is the present value of the Hubble parameter in units of 100 kms^{-1}Mpc^{-1}. We take $\Omega_b h^2 = 2.273 \pm 0.062$, the 5-year mean value of WMAP (Komatsu et al. 2008), and obtain:

$$6.049 \cdot 10^{-10} \leq \eta \leq 6.403 \cdot 10^{-10}.$$

Implementing η and $\tau_n = 878.5$ s (Serebrov et al. 2005), the numerical code delivers

$$Y_P = 0.2467 \pm 0.0003$$

and thus eq. (57) constrains the possible time variation of v:

$$\left|\frac{\Delta v}{v}\right| \leq 1.5 \cdot 10^{-4}.$$

Using the observation based results of Izotov et al. (2007) we derive

$$Y_P = 0.2516 \pm 0.0011 \quad \Rightarrow \quad \left|\frac{\Delta v}{v}\right| \leq 5.6 \cdot 10^{-4}.$$

and the results of Peimbert et al. (2007) lead to

$$Y_P = 0.2477 \pm 0.0029 \quad \Rightarrow \quad \left|\frac{\Delta v}{v}\right| \leq 1.4 \cdot 10^{-3}.$$

We avoid the term "observational results" because the cited publications more or less consist of interpretation of observational ^{4}He abundance plus theoretical input and constraints by the cosmic microwave background. Especially the different interpretation as a result of the deficiently understood systematics lead to incompatible data. For consistency, we only cite data based on recent He I recombination coefficients by Porter et al. (2005, 2007).

We thank D. Tekle and A. Bauer for help on the Kawano code and S. Winitzki and K. Butler for valuable comments on the manuscript.

REFERENCES

Abele, H., Astruc Hoffmann, M., Baeßler, S., Dubbers, D., Glück, F., Müller, U., Nesvizhevsky, V., Reich, J., & Zimmer, O. 2002, Phys. Rev. Lett. **88**, 21

Arenhövel, H., & Sanzone, M. 1991, Photodisintegration of the Deuteron: A Review of Theory and Experiment, Springer-Verlag

Bethe, H. A., & Longmire, C. 1950, Phys. Rev. **77**, 647

Chamoun, N., Landau, S. J., Mosquera, M. E., & Vucetich, H. 2007, J. Phys. G: Nucl. Part. Phys. **34**, 163

Dent, T., Stern, S., & Wetterich, Ch. 2007, Phys. Rev. D **76**, 063513

Descouvemont, P., Adahchour, A., Angulo, C., Coc, A., & Vangioni-Flam, E. 2004, At. Data Nucl. Data Tables **88**, 203

Dixit, V.V., & Sher, M. 1988, Phys. Rev. D **37**, 1097

Gasser, J., & Leutwyler, H. 1982, Phys. Rep. **87**, 77

Gaßner, J. M., & Lesch, H. 2008, Int. J. Theor. Phys. **47**, 438

Griffiths, D. 1987. Introduction to elementary particles, Wiley, Canada

Ichikawa, K., & Kawasaki, M. 2002, Phys. Rev. D **65**, 123511

Izotov, Y. I., Thuan, T. X., & Stasinska, G. 2007, ApJ **662**, 15

Kawano, L. 1992, Fermilab Report No. FERMILAB-PUB-92, 004-A

Komatsu, E., Dunkley, J., Nolta, M. R., Bennett, C. L., Gold, B., Hinshaw, G., Jarosik, N., Larson, D., Limon, M., Page, L., Spergel, D. N., Halpern, M., Hill, R. S., Kogut, A., Meyer, S. S., Tucker, G. S., Weiland, J. L., Wollack, E., & Wright, E. L. 2008, arXiv:0803.0547v1

Kujat, J., & Scherrer, R. J. 2000, Phys. Rev. D **62**, 023510

Lacombe, M., Loiseau, B., Richard, J. M., Vinh Mau, R., Côté, J., Pirès, P., & de Tourreil, R. 1980, Phys. Rev. C **21**, 861

Landau, S. J., Mosquera, M. E., & Vucetich, H. 2006, ApJ, **637**, 38

Li, B., & Chu, M.-C. . 2006, Phys. Rev. D **73**, 023509

Machleidt,R., Holinde, K., & Elster, Ch. 1987, Phys. Rep. **149**, 1

Müller, C. M., Schäfer, G., & Wetterich, C. 2004, Phys. Rev. D **70**, 083504

Mukhanov, V. 2004, Int. J. Theor. Phys. **43**, 669

Peimbert, A., Luridiana, V., & Peimbert, A. 2007, ApJ **666**, 636

Porter, R. L., Bauman, R. P., Ferland, G. J, & MacAdam, K. B. 2005, ApJ, **622**, L73

Porter, R. L., Ferland, G. J., & MacAdam, K. B. 2007, ApJ, **657**, 327

Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics, Wiley, Canada

Scherrer, R. J., & Spergel, D. N. 1993, Phys. Rev. D **47**, 4774

Serebrov, A., Varlamov, V., Kharitonov, A., Fomin, A., Pokotilovski, Yu., Geltenbort, P., Butterworth, J., Krasnoschekova, I., Lasakov, M., Taldaev, R., Vassiljev, A., & Zherebtsov, O. 2005, Phys. Lett. B **605**, 72-78

Steigman, G. 2008, astro-ph/0606206v3

Uzan, J.P. 2003, Rev. Mod. Phys. **75**, 403

Wagoner, R. V. 1973, ApJ, **179**, 343 (code available for public download at http://www-thphys.physics.ox.ac.uk/users/SubirSarkar/bbn.html)

Wiringa, R. W., Smith, R., & Ainsworth, T. L. 1984, Phys. Rev. C **29**, 1207

Yao, W.-M., et al. 2006, J. Phys. G **33**, 1 (2006) and 2007 partial update for the 2008 edition available on PDG www pages (URL: http://pdg.lbl.gov/).

Yoo, J. J., & Scherrer, R. J. 2003, Phys. Rev. D **67**, 043517

This preprint was prepared with the AAS LaTeX macros v5.2.